Indian Statistical Institute, Bangalore
M.Math II year 2018-2019
Semester II : Operator Theory

Mid-Sem Exam — Date: 23.02.2019
Maximum Marks: 80 — Duration: 3 hours

Note: Any score above 80 will be taken as 80. State the results very clearly that you are using in your answers.

1. (15) . Let $\mathcal{H}$ be a Hilbert space of dimension greater than 1. Show that the Banach space $\mathcal{B}(\mathcal{H})$ of bounded operators on $\mathcal{H}$, with operator norm, is not a Hilbert space.

2. (15) If $T \in \mathcal{B}(\mathcal{H})$, show that T is compact iff $|T|$ is compact.

3. $(5+10+15)$ Let H be a complex Hilbert space having an orthonormal basis $e_1, e_2, e_3, \ldots$. Let T be an operator satisfying $Te_n = a_n e_{n+1}$ for every $n = 1, 2, \ldots$.

   (a) State the conditions under which $T \in \mathcal{B}_0(\mathcal{H})$.

   (b) If $T \in \mathcal{B}(\mathcal{H})$, find $||T||$ and show that the spectrum of T is rotation invariant.

   (c) If $a_n = 1$ for all n, then for any given polynomial p, find the spectrum of $p(T)$ in terms of the roots of p.

4. $(5+10+10)$ Let $f \in \mathcal{C}(\mathbb{T})$ be given by $f(z) = (3 + z + \bar{z})z$. Let T_f be the Toeplitz operator given by f.

   (a) Show that T_f is not compact.

   (b) Show that T_f is a Fredholm operator and find its index.

   (c) Find the spectrum of T_f.